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COMPUTATIONAL ANALYSIS OF UNSTEADY FLOW OF BLOOD AND HEAT TRANSFER THROUGH A STENOSESD ARTERY IN A THIRD GRADE FLUID MODEL WITH SLIP CONDITIONS

Jimoh A[1], Okedayo G.T[2] & Ikpakyegh L.N[3]

[1]Research Scholar, Department of Mathematical Sciences, Kogi State University, Anyigba, Nigeria

[2]Research Scholar, Department of Mathematical Sciences, Ondo State University of Science and Technology Okitipupa, Ondo State, Nigeria

[3]Research Scholar, Department of Mathematics/Statistics/Computer Sciences, Federal University of Agriculture, Makurdi, Benue State, Nigeria

ABSTRACT

This paper investigates the computational analysis of unsteady flow of blood and heat transfer through a stenosed artery in a third grade fluid model with slip conditions. Incorporated into the model is an externally applied magnetic field. The solutions of the unsteady non-linear dimensionless momentum and energy equations are obtained using Galerkin's weighted residual and Fourth order Runge-Kutta methods. Effects of slip velocity, magnetic field, shear thinning, shear thickening and other parameters on the flow and heat transfer characteristics are presented graphically and discussed.

KEYWORDS: *Unsteady Blood Flow, Unsteady Heat Transfer, Magnetic Field, Slip Conditions, Stenosed Artery, Shear Thinning, Shear Thickening Third Grade Fluid And Galerkin's Weighted Residual Methods*

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INTRODUCITON

The study of blood flow in the cardiovascular system has generated a lot of interest, hence, the huge amount of literatures are studied on the subjects. Sanjeev and Chandrashkhar [1] considered mathematical model of power law fluid with an application of blood flow through an artery with stenosis. Their results showed that, increases in the size of the stenosis lead to increase in the pressure drop and flux but decrease the resistance to flow and wall shear stress. They concluded that the non-Newtonian behavior of blood is helpful in the functioning of diseased arterial circulation. The results concerning the velocity and temperature field of Newtonian blood flow in the presence of applied magnetic field was presented by Kumar and Diwakar [2]. They found out that the rate of heat transfer indicates the presence of magnetic field appreciably influenced by the application of the magnetic field. Dharmendra [3] presented a theoretical investigation to examine some of the significant characteristics of three layered oscillatory blood flow through stenosed arteries. Their results revealed that the size of intermediate and peripheral layers reduces in the expanded region with the increasing viscosity of fluid in peripheral layer, whereas, the opposite effect is observed for the viscosity of fluid in the intermediate layer. The pulsatile flow of the blood as two fluid models with the suspension of the erythrocyte in the core peripheral of plasma as a Newtonian fluid was investigated by Devajyoti and Uday [4]. From their finding, increased slip velocity lead to increase in the flow velocity and flow rate but decreased the effective viscosity. Effect of slip velocity on blood flow through a catheterized artery was investigated by Narendra et al [5]. They assumed that, the flowing blood is represented by a Newtonian fluid. From their analysis, they concluded that the slip velocity reduces the effective viscosity and wall shear stress.

Furthermore, Singh *et al*. [6] theoretically investigated the influence of externally imposed periodic body acceleration of the flow of blood through a time dependent stenosed arterial segment by taking into account of the slip velocity at the wall of the artery. The effect of slip velocity on blood flow through an arterial tube in the presence of multiple stenosis was studied by Arun [7]. He considered the effects of length of stenosis and shape parameter on resistance to flow and shear stress. He observed from the graphs that, the parameters have small variation for different values of stenosis shape parameter.

Several other researchers Bhatnagar and Strivastava [8], Amit and Shrivastav*et al* [11], Raja and Varshney [12] to mention but a few have in one way or the other modeled and studied the flow of blood through a constricted artery under the influence of magnetic field and slip conditions. None of these researchers considered the unsteady blood flow and heat transfer models in their studies.

However, studies on unsteady blood flow and unsteady heat transfer have received very scanty attention. In unsteady fluid flow or heat transfer, the conditions at any point of the flow or heat transfer are time dependent. Thus, the governing equations describing the fluid flow or heat transfer become cumbersome to handle as the equations now have time derivative. Some of the researchers investigated unsteady blood flow model without considering the heat flow includes Geeta and Siddique [13], Mandal [14], Islam [15], Srikanth et al [16] and Ikbal*et al* [17].

Therefore, in this study, we analyzed the unsteady blood flow and heat transfer through a stenosed artery in a third fluid model with slip velocity at the constricted artery. The presence of externally applied magnetic field is also taking into consideration in this study.

Mathematical Models

The momentum equation describing the unsteady fluid flow and unsteady heat transfer as obtained by Mohammed [18] are respectively given as

$$\frac{\partial w}{\partial t} = \frac{\mu}{\rho}\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) + \frac{6\beta_3}{\rho}\left(\frac{\partial w}{\partial r}\right)^2 \frac{\partial^2 w}{\partial r^2} + \frac{2\beta_3}{\rho r}\left(\frac{\partial w}{\partial r}\right)^2 + \frac{\alpha_1}{\rho r}\frac{\partial^2 w}{\partial r \partial t} + \frac{\alpha_1}{\rho}\frac{\partial^3 w}{\partial r^2 \partial t} - \frac{\partial \hat{P}}{\rho \partial z} - \frac{\sigma \beta_0^2}{\rho} w \tag{2.1}$$

$$\frac{\partial T}{\partial t} = \frac{\mu}{\rho c_p}\left(\frac{\partial w}{\partial r}\right)^2 + \frac{\alpha_1}{\rho c_p}\left(\frac{\partial^2 w}{\partial r \partial t}\right)\frac{\partial w}{\partial r} + \frac{2\beta_3}{\rho c_p}\left(\frac{\partial w}{\partial r}\right)^4 + \frac{K}{\rho c_p}\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) \tag{2.2}$$

From the right hand side of equation (2.1), the first term can be written as

$$\frac{\mu}{\rho}\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) = \frac{\mu}{\rho r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) \tag{2.3}$$

while that of equation (2.2) from the right hand side, the third term can be written as

$$\frac{K}{\rho C_\rho}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)=\frac{K}{r\rho C_\rho}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{2.4}$$

When we substituted (2.3) into (2.1) and (2.4) into (2.2), we respectively obtained

$$\frac{\partial w}{\partial t}=\frac{\mu}{\rho r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)+\frac{6\beta_3}{\rho}\left(\frac{\partial w}{\partial r}\right)^2\frac{\partial^2 w}{\partial r^2}+\frac{2\beta_3}{\rho r}\left(\frac{\partial w}{\partial r}\right)^2+\frac{\alpha_1}{\rho r}\frac{\partial^2 w}{\partial r\partial t}+\frac{\alpha_1}{\rho}\frac{\partial^3 w}{\partial r^2\partial t}-\frac{\partial \hat{p}}{\rho\partial z}-\frac{\sigma\beta_0^2}{\rho}w \tag{2.5}$$

$$\frac{\partial T}{\partial t}=\frac{\mu}{\rho c_p}\left(\frac{\partial w}{\partial r}\right)^2+\frac{\alpha_1}{\rho c_p}\left(\frac{\partial^2 w}{\partial r\partial t}\right)\frac{\partial w}{\partial r}+\frac{2\beta_3}{\rho c_p}\left(\frac{\partial w}{\partial r}\right)^4+\frac{K}{\rho c_p}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{2.6}$$

Since we employed slip velocity in the constricted artery as shown in Figure 1, the corresponding slip conditions to equations (2.5) and (2.6) are respectively given as

$$\left.\begin{array}{lll} w=w_s & \text{at} & r=R(z) \\ \frac{\partial w}{\partial r}=0 & \text{at} & r=0 \end{array}\right\} \tag{2.7}$$

and

$$\left.\begin{array}{lll} T=T_w & \text{at} & r=R(z) \\ \frac{\partial T}{\partial r}=0 & \text{at} & r=0 \end{array}\right\} \tag{2.8}$$

To non-dimensionalize equations (2.5), (2.6), (2.7) and (2.8), we introduce the following parameters and variables.

$$\left.\begin{array}{ll} \bar{w}=\frac{w}{d/t_0}, & y=r/R_0 \\ \bar{t}=\frac{t}{t_0}, & V_{01}=\frac{w_s t_0}{d} \end{array}\right\} \quad \bar{\theta}=\frac{T-T_w}{T_m-T_w} \tag{2.9}$$

When equation (2.9) is substituted into (2.5) and (2.6), after simplifying we obtained respectively

$$\frac{\partial\bar{w}}{\partial\bar{t}}=\frac{1}{\mathrm{RE}_1}\cdot\frac{\partial}{\partial y}\left(y\frac{\partial\bar{w}}{\partial y}\right)+\Omega\left(6\left(\frac{\partial\bar{w}}{\partial y}\right)^2\frac{\partial^2\bar{\omega}}{\partial y^2}+\frac{2}{y}\left(\frac{\partial\bar{w}}{\partial y}\right)^3\right)+\Omega_1\left(\frac{1}{y}\frac{\partial^2\bar{w}}{\partial y\partial\tau}+\frac{\partial^3\bar{w}}{\partial y^2\partial\bar{t}}\right)+\mathrm{G}_1-\mathrm{M}_1\bar{w} \tag{2.10}$$

as the dimensionless momentum equation.

where

$$\left.\begin{array}{l} \mathrm{RE}_1=\frac{R_0^2}{Vt_0},\ \mathrm{G}_1=-\frac{t_0^2}{d\rho}\frac{\partial\hat{p}}{\partial z},\ \Omega=\frac{\beta_3 d^2}{t_0\rho R_0^4} \\ \Omega_1=\frac{\alpha_1}{R_0^2\rho},\quad \mathrm{M}_1=\frac{t_0\sigma\beta_0^2}{\rho} \end{array}\right\} \tag{2.11}$$

with corresponding dimensionless slip conditions simplified as

$$\left.\begin{array}{lll} \bar{w}=V_{01} & \text{at} & y=\frac{R}{R_0}=R_b \\ \frac{\partial\bar{w}}{\partial y}=0 & \text{at} & y=0 \end{array}\right\} \tag{2.12}$$

And

$$\frac{\partial\bar{\theta}}{\partial\bar{t}} = E_{n1}\left(\frac{\partial\bar{w}}{\partial y}\right)^2 + \phi\left(\frac{\partial\bar{w}}{\partial y}\right)^4 + \phi_1\left(\frac{\partial^2\bar{w}}{\partial y\partial\bar{t}}\right)\left(\frac{\partial\bar{w}}{\partial y}\right) + \Lambda_1\frac{1}{y}\frac{\partial}{\partial y}\left(y\frac{\partial\bar{\theta}}{\partial y}\right) \quad (2.13)$$

as the dimensionless energy equation.

where

$$\left.\begin{aligned} E_{n1} &= \frac{Vd^2}{t_0(T_m-T_w)R_0^2C_\rho}, \qquad \Phi = \frac{2\beta_3 d^4}{t_0^3(T_m-T_w)R_0^4\rho C_\rho} \\ \phi_1 &= \frac{\alpha_1 d^2}{t_0^2R_0^2\rho C_\rho(T_m-T_w)} \qquad \text{and} \qquad \Lambda_1 = \frac{Kt_0}{R_0^2\rho C_\rho} \end{aligned}\right\} \quad (2.14)$$

with the corresponding dimensionless slip conditions simplified as

$$\left.\begin{aligned} \bar{\theta} &= 0 \quad \text{at} \quad y = R_b \\ \frac{\partial\bar{\theta}}{\partial y} &= 0 \quad \text{at} \quad y = \end{aligned}\right\} \quad (2.15)$$

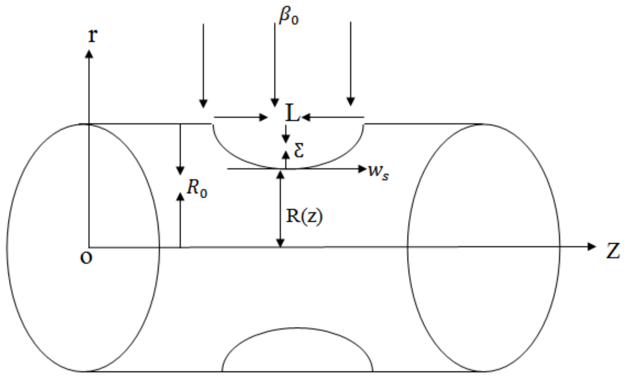

Figure 1: Geometry of the Stenosis.

and has been described by Young [19] and Biswas [20]

$$\left.\begin{aligned} \frac{R(z)}{R_0} &= 1 - \frac{\varepsilon}{2R_0}\left[1 + \frac{cos\pi z}{L}\right], \quad \text{for} \quad |z|\leq L \\ R_0&, \quad \text{for} \quad |z|>L \end{aligned}\right\} \quad (2.16)$$

Methods of Solution

We have used Galerkin weighted residual method for getting the required solutions of the problems. By this method, to obtain the velocity profile of the unsteady fluid flow, we assume a solution of the form

$$\bar{w}(y,t) = a_0(t) + a_1(t)y + a_2(t)y^2 \quad (3.1)$$

Subjecting (3.1) to the slip conditions (2.12) and after simplifying yields

$$\bar{w}(y,t) = \frac{V_{01}y^2}{Rb^2} + a_0(t)\left(1 - \frac{y^2}{Rb^2}\right) + a_2(t)\frac{y^2}{Rb^2}\left(1 - \frac{y^2}{Rb^2}\right) \quad (3.2)$$

Now,

$$\text{Let } \bar{r} = \frac{y}{Rb} \quad (3.3)$$

Putting (3.4) into (3.2) to obtain

$$w(\bar{r},t) = V_{01}\bar{r}^2 + a_0(t)(1-\bar{r}^2) + a_2(t)\bar{r}^2(1-\bar{r}^2) \tag{3.4}$$

For convenience sake, we drop the bar and write (3.4) as

$$w(r,t) = V_{01}r^2 + a_0(t)(1-r^2) + a_2(t)r^2(1-r^2) \tag{3.5}$$

The residual for equation (2.10) can be written as

$$RR_1\big(r,a_0(t),a_2(t)\big) = \frac{\partial w}{\partial t} - G_1 - \frac{1}{REI}\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) - \Omega\left(6\left(\frac{\partial w}{\partial r}\right)^2\frac{\partial^2 w}{\partial r^2} + \frac{2}{r}\left(\frac{\partial w}{\partial r}\right)^3\right) - \Omega_1\left(\frac{1}{r}\frac{\partial^2 w}{\partial r\partial t} + \frac{\partial^3 w}{\partial r^2\partial t}\right) + M_1 w$$

(3.6)

substituting (3.5) into (3.6) and simplifying we obtain

$RR_1\big(r,a_0(t),a_2(t)\big) = -G - \frac{1}{REI}(4V_{01} + 4a_2(t) - 4a_0(t) - 16a_2(t)r^2) + \dot{a}_0(t)(1-r^2) + \dot{a}_2(t)r^2(1-r^2) +$
$4\Omega_1\big(\dot{a}_0(t) - \dot{a}_2(t) + 4r^2\dot{a}_2(t)\big) - (48V_{01}^2r^2 - 144V_{01}^2a_0(t)r^2 + 144V_{01}^2a_2(t)r^2 - 480V_{01}^2a_2(t)r^4 + 144V_{01}a_0^2(t)r^2 -$
40V01a0ta2tr2+960V01a0ta2tr4+144V01a22tr2−144V01a22tr44V01a22tr6+144a02a2tr2−480a02ta2tr4−144a0ta22tr2+960a0ta22tr4−1344a0ta22tr6−480a23tr4+1344a23tr6−48a03tr2+48a23tr2−1152a23tr8−128a23tr8+192V01a22tr6−192a0ta22tr6+192a23tr6−96V012a2tr4+192V01a0ta2tr4−192V01a22tr4−96a02ta2tr4+192a0ta22tr4−96a23tr4+16V013r2−48V012a0tr2+48V01a02tr2−96V01a0ta2tr2+48V01a22tr2−16a03t+48a02ta2tr2−48a0ta22tr2+16a23tr2−M1V01r2+a0t−a0tr2+a2tr2−a2tr4

(3.7)

By differentiating (3.5) with respect to $a_0(t)$ and $a_2(t)$ we obtain the weight functions respectively as

$$U_1(r) = (1-r^2) \tag{3.8}$$

and

$$U_2(r) = r^2(1-r^2) \tag{3.9}$$

We obtained the following systems by taking the orthogonality of the residue $RR_1(a_0(t),a_2(t),r)$ with respect to the weight functions $U_1(r)$and $U_2(r)$

$$\int_0^1 (1-r^2)\, RR_1(a_0(t),a_2(t),r)dr = o \tag{3.10}$$

$$\int_0^1 r^2(1-r^2)\, RR_1(a_0(t),a_2(t),r)dr = 0 \tag{3.11}$$

When equation (3.7) is substituted into (3.10) and (3.11) respectively, after evaluating and simplifying in full we obtain the following systems

$-\frac{2G_1}{3} + \frac{4}{REI}\left(-\frac{2V_{01}}{3} + \frac{2a_0(t)}{3} - \frac{2a_2(t)}{15}\right) + \frac{8\dot{a}_0(t)}{15} + \frac{8\dot{a}_2(t)}{105} + 4\Omega_1\left(\frac{2\dot{a}_0(t)}{3} - \frac{2\dot{a}_2(t)}{15}\right) - 16\Omega\left(-\frac{2V_{01}^2}{5} - \frac{32V_{01}^2a_0(t)}{5} -\right.$
16V012a2t35−8V01a02t5−362V01a22t105−278V01a0ta2t105+16a02ta2t35+56a0ta22t105+1475a23t3465+8a03t15+M1−2V0115−8a0t15−8a2t105=0

(3.12)

and

$-\frac{2G_1}{15}+\frac{4}{REI}\left(-\frac{2V_0}{15}+\frac{2\dot{a}_0(t)}{15}+\frac{2\dot{a}_2(t)}{21}\right)+\frac{8\dot{a}_0(t)}{105}+\frac{8\dot{a}_2(t)}{315}+4\Omega_1\left(\frac{2\dot{a}_0(t)}{15}+\frac{2\dot{a}_2(t)}{21}\right)-16\Omega\left(-\frac{6V_0^2}{35}+\frac{54V_0^2a_2(t)}{35}+\right.$
16V02a2t35−24V0a2t35−1107V0a2t1155−58V0a0ta2t35−334a02ta2t35−896a0ta22t385+33836a23t5005+8a03t35−M1−2V035−8a0t105−3a2t315=0.

(3.13)

Where denotes differentiation with respect to t.

Equations (3.12) and (3.13) can be re-written respectively as

$$(1848+9240\Omega_1)\dot{a}_0(t)+(264-1848\Omega_1)\dot{a}_2(t)+29568\Omega a_0^3(t)+23600\Omega a_2^3(t)-88704\Omega V_{01}a_0^2(t)-191136\Omega V_{01}a_2^2(t)+29568\Omega a_0(t)a_0^2(t)+25344\Omega a_2(t)a_0^2(t)-146784\Omega V_{01}a_0(t)a_0^2(t)-\left(1848M_1+354816\Omega V_{01}^2-\frac{9240}{REI}\right)a_0(t)-\left(264M_1-25344\Omega V_{01}^2-\frac{1848}{REI}\right)a_2(t)=2310G_1+\frac{9240V_{01}}{REI}+22176\Omega V_{01}^2+462M_1V_{01} \quad (3.14)$$

and

$$(3432+24024\Omega_1)\dot{a}_0(t)+(1144+17160\Omega_1)\dot{a}_2(t)+164736\Omega a_0^3(t)+4872384\Omega a_2^3(t)-494208\Omega V_{01}a_0^2(t)-690768\Omega V_{01}a_0^2(t)-6877728\Omega a_0^2(t)a_2(t)-1677312\Omega a_2^2(t)a_0(t)-1194336\Omega V_{01}a_0(t)a_2(t)-\left(3432M_1-1111968\Omega V_{01}^2+\frac{24024}{REI}\right)a_0(t)-\left(429M_1-329472\Omega V_{01}^2+\frac{17160}{REI}\right)a_2(t)=6006G_1+\frac{24024}{REI}+123552\Omega V_{01}^2+2574M_1V_{01} \quad (3.15)$$

By substituting the appropriate values of the parameters G_1, V_{01}, REI, Ω, Ω_1, $M_1$and t into (3.14) and (3.15), after simplifying we obtain

$$-85.33333333a_0^3(t)-14.77633478a_2^3(t)+64.000000a_0^2(t)+17.0666666a_2^2(t)-21.33333333a_0^2(t)a_2(t)-85.33333333a_2^2(t)a_0(t)+36.57142858(t)a_2(t)-19.14962963a_0(t)-4.005502645a_2(t)-2.666666667\dot{a}_0(t)+0.5333333333\dot{a}_2(t)=-3.062407407 \quad (3.16)$$

and

$$-36.57142857a_0^3(t)-15.00366300a_2^3(t)+27.42857142a_0^2(t)+13.57575758a_2^2(t)-73.14285714a_0^2(t)a_2(t)-54.30303030a_2^2(t)a_0(t)+36.57142858a_0(t)a_2(t)-7.476402115a_0(t)-5.003597883a_2(t)-0.5333333333\dot{a}_0(t)+0.3809523869\dot{a}_2(t)=-0.9145767196 \quad (3.17)$$

Solving the system of non-linear first order ordinary differential equations (3.16) and (3.17) with initial conditions $a_0(0)=0$, $a_2(0)=0$ and time t = 0.5 using fourth order Runge–Kutta method we obtain the values for$a_0(t)$ and $a_2(t)$ and when submitted into (3.5) we obtain the velocity profile for the unsteady fluid flow of blood as shown in Table1 for the various values of the parameters.

Similarly, to obtain the temperature profile for the unsteady heat transfer using the Gerlakin's method, we assume a solution of the form

$$\overline{\theta}(y,t)=a_0(t)+a_3(t)y+a_4(t)y^2 \quad (3.18)$$

Subjecting (3.18) to the slip conditions (2.15) and after simplifying we obtain

$$\overline{\theta}(y,t) = a_3(t)\left(1-\frac{y^2}{R_b^2}\right) + a_4(t)\frac{y^2}{R_b^2}\left(1-\frac{y^2}{R_b^2}\right) \tag{3.19}$$

By using the transformation (3.3) and dropping bar, equation (3.19) can be written as

$$\theta(r,t) = a_3(t)(1-r^2) + a_4(t)r^2(1-r^2) \tag{3.20}$$

The residual for the unsteady heat transfer can be written as

$$RR_2\big(r, a_3(t), a_4(t)\big) = \frac{\partial\theta}{\partial \mathrm{t}} - E_{n1}\left(\frac{\partial w}{\partial r}\right)^2 - \phi\left(\frac{\partial w}{\partial r}\right)^4 - \phi_1\left(\frac{\partial^2 w}{\partial r\partial \mathrm{t}}\right)\left(\frac{\partial w}{\partial r}\right) - \Lambda_1\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right) \tag{3.21}$$

By substituting (3.5) and (3.20) into (3.21), after simplifying we obtain

$RR_2\big(r, a_3(t), a_4(t)\big) = a_3(t)(1-r^2) + a_4(t)r^2(1-r^2) - E_{n1}(4V_{01}^2r^2 - 8a_0(t)V_{01}r^2 + 8a_2(t)V_{01}r^2 -$
16V01a2tr2+4a02tr2− 8a0ta2tr2+16a0ta2tr4+4a22tr2− 16a22tr4+16a22tr6− ϕ64V013a2tr2− 64V013a0tr4 − 64V013a2tr4− 128V013a2tr6+96V012a22tr4+96V012a02tr4− 384V012a22tr6+384V012a22tr8− 512V01a23 tr10− 384V01a23tr6− 64V01a03tr4+768V01a23tr8+64V01a23tr4− 64a03ta2tr4+128a03ta2tr6+96a02ta22t r4− 384a02ta22tr6+384a02ta22tr8+512a0ta23tr10+384a0ta23tr6− 768a0ta23tr8− 64a0ta23tr4+16a23tr4 − 128a23r6+16a04r4− 512a24r10+384a24r8+16V014r4+256a24r12+192V01a02a2r4− 384V01a02a2r6− 192 V01a0a22r4+768V01a0a22r6− 768V01a0a22r8− 192V012a0a2r4− ϕ1− 4V01ra0t+4V01ra2t− 8r4V01a2t+4r2 a0ta0t− 4r2a0ta2t+8r4a0ta2t− 4r2a2ta0t+4r2a2ta2t− 8r4a2ta2t+8r4a2ta0t− 8r4a2ta2t+16r6a2ta2t− Λ14a 3t− 4a4t+16a4tr2 (3.22)

By replacing $RR_1(a_0(t), a_2(t), r)$ with $RR_2(a_3(t), a_4(t), r)$ in (3.10) and (3.11), after evaluating and simplifying in full we obtained the following system of equations

$\frac{8\dot{a}_3(t)}{15} + \frac{8\dot{a}_4(t)}{105} + 4E_{n1}\left(-\frac{2V_{01}^2}{15} + \frac{4V_{01}a_0(t)}{15} + \frac{60V_{01}a_2(t)}{105} - \frac{-2a_0^2(t)}{15} + \frac{4a_0(t)a_2(t)}{105} - \frac{6a_2^2(t)}{189}\right) + 16\phi\left(-\frac{64V_{01}^3a_2(t)}{315} +\right.$
8V013a0t35− 228V012a22t3465− 12V012a02t35+504V01a23t45045+8V01a23t35− 8a2ta03t315− 56a0ta23t5 005− 228a02ta22t3465− 2a04t35− 3090a24t675675+8v0a2ta02t105+152V01a0ta22t1155+24V0124a0ta2t35 +4ϕ1V01a0t4− 51V01a2t140− 6a2ta2t189+2a0ta2t105− 68a2ta0t105+4Λ− 2a3t3+2a4t15=0
(3.23)

and

$\frac{8\dot{a}_3(t)}{105} + \frac{8\dot{a}_4(t)}{315} + 4E_{n1}\left(-\frac{2V_{01}^2}{35} + \frac{4V_{01}a_0(t)}{35} + \frac{4V_{01}a_2(t)}{315} - \frac{2a_0^2(t)}{315} + \frac{4a_0(t)a_2(t)}{315} - \frac{38a_2^2(t)}{3465}\right) + 16\phi\left(-\frac{64V_{01}^3a_2(t)}{693} +\right.$
8V013a0t63− 372V012a22t9009− 12V012a02t63+1992V01a23t135135+8V01a23t63− 8a2ta03t231− 1992a0ta 23t135135− 124a02ta22t3003− 2a04t63− 7074a24t297295+22v0a2ta02t6935+744V01a0ta22t9009+24V012 4a0ta2t63+4ϕ1V01a0t12− 222V01a2t1512− 86a2ta2t1155+2a0ta2t315− 24a2ta0t315+4Λ− 2a3t15− 10a4t10 5=0 (3.24)

Equations (3.23) and (3.24) can be re-written respectively as

$675675\phi V_{01}\dot{a}_0(t) - 984555\phi V_{01}\dot{a}_2(t) + 51480\phi V_{01}\dot{a}_2(t) - 1750320\phi_1 a_2(t)\dot{a}_0(t) -$
$85800\phi_1 a_2(t)\dot{a}_2(t) + 617760\phi a_0^4(t) - 2087835750\phi a_2^4(t) + 2471040\phi V_{01}a_0^3(t) + 120960\phi V_{01}a_2^3(t) -$

$$120960\phi a_0(t)a_2^3(t) - 274560\phi a_2(t)a_0^3(t) + 1422720\phi V_{01}a_0(t)a_2^2(t) + 823680\phi V_{01}a_2(t)a_0^2(t) - 711360\phi a_0^2(t)a_2^2(t) - (360360E_{n1} + 3706560\phi V_{01}^2)a_0^2(t) - (85800E_{n1} + 711360\phi V_{01}^2)a_2^2(t) + (720720E_{n1}V_{01} + 2471040\phi v V_{01}^3)a_0(t) + (1544400E_{n1}V_{01} - 10810800\phi V_{01}^3)a_2(t) + (1544400E_{n1} + 7413120\phi V_{01}^2)a_0(t)a_2(t) + (360360 - 1801800\Lambda)\dot{a}_3(t) + (51480 + 360360\Lambda)\dot{a}_4(t) = 360360E_{n1}V_{01}^2 \quad (3.25)$$

and

$$765765\phi V_{01}\dot{a}_0(t) - 1349205\phi_1\dot{a}_2(t) - 58344\phi_1 a_0(t)\dot{a}_2(t) + 3734016\phi_1 a_2(t)\dot{a}_0(t) - 684216\phi_1 a_2(t)\dot{a}_2(t) - 1166880\phi a_0^4(t) - 113184\phi a_2^4(t) + 4667520\phi v V_{01}a_0^3(t) + 541824\phi V_{01}a_2^3(t) - 541824\phi a_0(t)a_2^3(t) - 1272960\phi a_2(t)a_0^3(t) + 1166880\phi V_{01}a_2(t)a_0^2(t) + 3035520\phi V_{01}a_0(t)a_2^2(t) - 1517760\phi a_0^2(t)a_2^2(t) - (525096E_{n1} + 7001280\phi V_{01}^2)a_0^2(t) - (100776E_{n1} + 1517760\phi V_{01}^2)a_2^2(t) + (1050192E_{n1}V_{01} + 4667520\phi V_{01}^3)a_0(t) + (116688E_{n1}V_{01} - 3394560\phi V_{01}^3)a_2(t) - (116688E_{n1} + 14002560\phi V_{01}^2)a_0(t)a_2(t) + (175032 - 1225224\Lambda)a_3(t) + (58344 - 875160\Lambda)a_4(t) = 525096E_{n1}V_{01}^2 \quad (3.26)$$

Substituting the appropriate values of the parameters ϕ, ϕ_1, V_{01}, E_{n1} and Λ_1 into (3.25) and (3.26) respectively, we obtain

$$0.1066666667\dot{a}_0(t) - 0.01523809524\dot{a}_2(t) + 0.53333333\dot{a}_3(t) + 0.0761904769\dot{a}_4(t) + 0.076190476\dot{a}_0(t)a_2(t) + 0.076190476\dot{a}_2(t)a_0(t) + 0.53333333\dot{a}_0(t)a_0(t) - 0.126984127\dot{a}_2(t)a_2(t) - 54.857142857a_0^4(t) - 4.3902763903a_2^4(t) + 43.88571429a_0^3(t) + 2.1148251748a_2^3(t) + 24.38097381a_0^3(t)a_2(t) - 10.741258741a_0(t)a_2^3(t) - 13.96571429a_0^2(t) + 14.62857143a_0^2(t)a_0^2(t) + 25.26753247a_0(t)a_2^2(t) - 2.697142857a_0(t)a_2(t) + 4.9333333a_3(t) - 0.98666666a_4(t) = 0.1197714286 \quad (3.27)$$

and

$$0.04571428571\dot{a}_0(t) + 0.005079365099\dot{a}_2(t) + 0.076190476\dot{a}_3(t) + 0.0253968254\dot{a}_4(t) - 0.0253968254\dot{a}_0(t)a_2(t) - 0.0253968254\dot{a}_2(t)a_0(t) - 0.2285714286\dot{a}_0(t)a_0(t) - 0.0438672439\dot{a}_2(t)a_2(t) - 30.476190476a_0^4(t) - 2.9561027208a_2^4(t) + 24.38095238a_0^3(t) + 2.830236430a_2^3(t) - 33.246753247a_0^3(t)a_2(t) - 14.151182151a_0(t)a_2^3(t) - 7.657142857a_0^2(t) - 1.651415252a_2^2(t) + 19.94805195a_0^2(t)a_2(t) + 15.85614386a_0(t)a_2^2(t) - 39.64035964a_0^2(t)a_2^2(t) + 1.112380952a_0(t) + 0.2812121212a_2(t) - 4.065800866a_0(t)a_2(t) + 0.98666666a_3(t) + 0.7047619048a_4(t)$$

$$= 0.06247619048 \quad (3.28)$$

Using fourth order Runge–kutta method, we solve the system of non-linear ordinary differential equations (3.16), (3.17), (3.27) and (3.28) with initial conditions $a_0(0) = 0, a_2(0) = 0, a_3(0) = 0, a_4(0)=0$ and time $t = 0$. When substituting the values of $a_3(t)$ and $a_4(t)$ into (3.20) after simplifying we obtain the temperature profile for the unsteady heat transfer as shown in Table 2 for various values of the parameters.

Volume Flow Rate

The volume flow rate denoted by Q is given by

$$Q = 2\pi \int_0^{R(z)} rw(r)dr \quad (3.29)$$

Putting (4.7) into (3.44) and evaluate to obtain

$$Q = 12\left[3V_0(R(z))^4 + a_0(t)\left(6(R(z))^2 - 3(R(z))^4\right) + a_2(t)\left(3(R(z))^4 - 2(R(z))^6\right)\right] \quad (3.30)$$

Shear Stress

The shear stress denoted by $\tau_S$is given as

$$\tau_s = \mu\frac{\partial w}{\partial r}\Big|_{r=R(z)} + 2\beta_3\left(\frac{\partial w}{\partial r}\right)^3\Big|_{r=R(z)} \quad (3.31)$$

Simplified (3.46) to obtain

$$\tau_s = 2\mu R(Z)\left(V_0 - a_0(t) + a_2(t) - 2R((Z))^2 a_2(t)\right) + 16R(Z)\beta_3(V_0 - a_0(t) + a_2(t) - 2(R(Z))^2 a_2) \quad (3.32)$$

Resistance to Flow

The resistance to flow can be denoted as ψ and is given by

$$\psi = \frac{-\frac{\partial \hat{P}}{\partial z}}{12\left[3V_0(R(z))^4 + a_0(t)\left(6(R(z))^2 - 3(R(z))^4\right) + a_2(t)\left(3(R(z))^4 - 2(R(z))^6\right)\right]} \quad (3.33)$$

Table 1: Values of the Parameters used in the Numerical Results and the Corresponding Velocity Profile for the Unsteady Blood Flow Model

Figures	G_1	V_{01}	REI	Ω	Ω_1	M_1	t	w(r, t)
2	1.5	0.25	0.9	10	1	0.35	0.5	$-0.1437r^2 + 0.3437 + 0.0335r^2(1-r^2)$
	2.0	0.25	0.9	10	1	0.35	0.5	$-0.1858r^2 + 0.3858 + 0.0296r^2(1-r^2)$
	2.5	0.25	0.9	10	1	0.35	0.5	$-0.2290r^2 + 0.4290 + 0.0262r^2(1-r^2)$
3	1.5	0.25	0.9	10	1	0.35	0.5	$-0.2261r^2 + 0.4261 + 0.0260r^2(1-r^2)$
	1.5	0.25	0.9	10	1	0.65	0.5	$-0.2174r^2 + 0.4174 + 0.0249r^2(1-r^2)$
	1.5	0.25	0.9	10	1	0.95	0.5	$-0.2090r^2 + 0.4090 + 0.0237r^2(1-r^2)$
4	1.5	0.25	0.9	10	1	0.35	0.5	$-0.0572r^2 + 0.2572 + 0.0009r^2(1-r^2)$
	1.5	0.35	0.9	10	1	0.35	0.5	$-0.1538r^2 + 0.3538 + 0.0019r^2(1-r^2)$
	1.5	0.45	0.9	10	1	0.35	0.5	$-0.2510r^2 + 0.4510 + 0.0030r^2(1-r^2)$
5	1.5	0.25	0.9	10	1	0.35	0.5	$-0.1246r^2 + 0.3246 + 0.0168r^2(1-r^2)$
	1.5	0.25	0.9	10	5	0.35	0.5	$-0.2016r^2 + 0.4016 + 0.0073r^2(1-r^2)$
	1.5	0.25	0.9	10	9	0.35	0.5	$-0.2542r^2 + 0.4542 + 0.0001r^2(1-r^2)$
6	1.5	0.25	0.9	10	1	0.35	0.5	$-0.3246r^2 + 0.5246 + 0.0499r^2(1-r^2)$
	1.5	0.25	0.9	20	1	0.35	0.5	$-0.3108r^2 + 0.5108 + 0.0538r^2(1-r^2)$
	1.5	0.25	0.9	30	1	0.35	0.5	$-0.2848r^2 + 0.4848 + 0.0554r^2(1-r^2)$
7	1.5	0.25	0.3	10	1	0.35	0.5	$-0.0053r^2 + 0.2053 + 0.000Ir^2(1-r^2)$
	1.5	0.25	0.6	10	1	0.35	0.5	$-0.0094r^2 + 0.2094 + 0.0001r^2(1-r^2)$
	1.5	0.25	0.9	10	1	0.35	0.5	$-0.0091r^2 + 0.2091 + 0.0005r^2(1-r^2)$

Table 2: Values of the Parameters used in the Numerical Results and the Corresponding Temperature Profile for the Unsteady Heat Transfer Model

Figures	ϕ	E_{n1}	Λ_1	V_{01}	Φ_1	θ(r, t)
8	1.25	1.5	1.35	0.25	2.0	$0.007 - 0.007r^2 + 0.001r^2(1-r^2)$
	1.50	1.5	1.35	0.25	2.0	$0.008 - 0.008r^2 + 0.013r^2(1-r^2)$
	1.75	1.5	1.35	0.25	2.0	$0.009 - 0.009r^2 + 0.015r^2(1-r^2)$
9	1.25	1.5	1.35	0.25	2.0	$0.006 - 0.006r^2 + 0.008r^2(1-r^2)$
	1.25	1.8	1.35	0.25	2.0	$0.007 - 0.007r^2 + 0.009r^2(1-r^2)$
	1.25	2.1	1.35	0.25	2.0	$0.008 - 0.008r^2 + 0.010r^2(1-r^2)$
10	1.25	1.5	1.35	0.25	2.0	$0.007 - 0.007r^2 + 0.008r^2(1-r^2)$
	1.25	1.5	1.65	0.25	2.0	$0.006 - 0.006r^2 + 0.006r^2(1-r^2)$
	1.25	1.5	1.95	0.25	2.0	$0.005 - 0.005r^2 + 0.005r^2(1-r^2)$

Table 2 Contd.,

11	1.25 1.25 1.25	1.5 1.5 1.5	1.35 1.35 1.35	0.25 0.25 0.25	1.0 2.0 3.0	$0.006 – 0.006r^2 + 0.008r^2(1-r^2)$ $0.010– 0.010r^2 + 0.001r^2(1-r^2)$ $0.013 – 0.013r^2 + 0.012r^2(1-r^2)$
12	1.25 1.25 1.25	1.5 1.5 1.5	1.35 1.35 1.35	0.25 0.35 0.45	2.0 2.0 2.0	$0.005 – 0.005r^2 + 0.006r^2(1- r^2)$ $0.006 –$ $0.006r^2 + 0.007r^2(1-r^2)$ $0.007 – 0.007r^2 + 0.008r^2(1-r^2)$

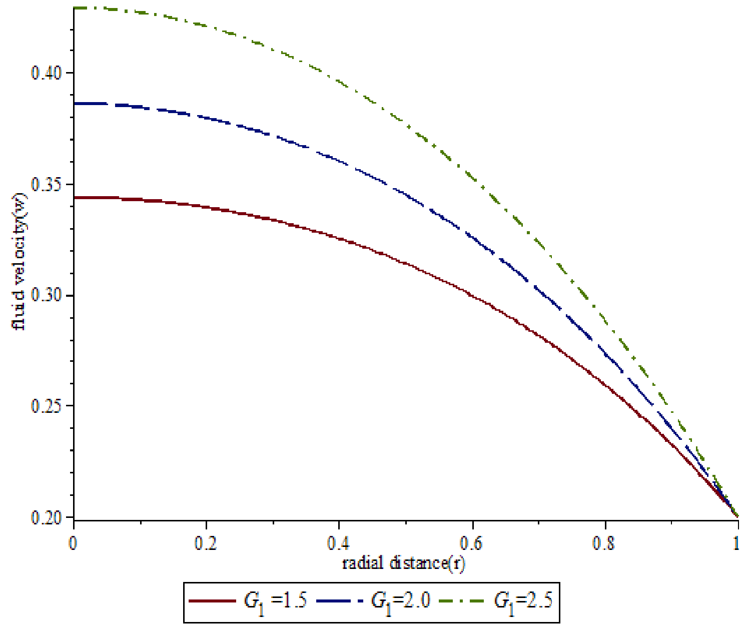

Figure 2: Variation of Velocity Profile of the Unsteady Blood Flow Model with Increasing Values of the Pressure Gradient in the Radial Direction.

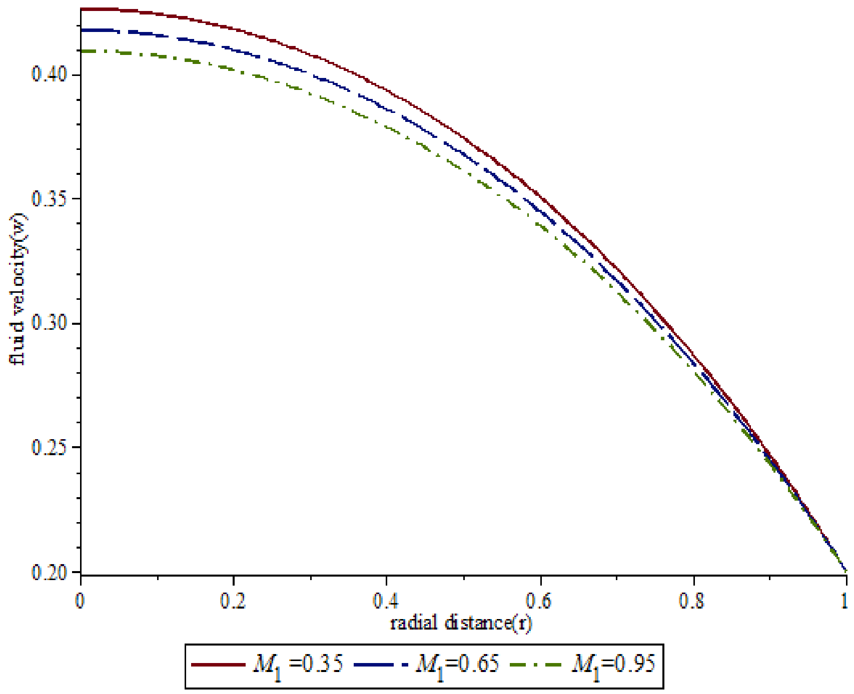

Figure 3: Variation of Velocity Profile of the Unsteady Blood Flow Model with Increasing Values of the Magnetic Field Parameter in the Radial Direction.

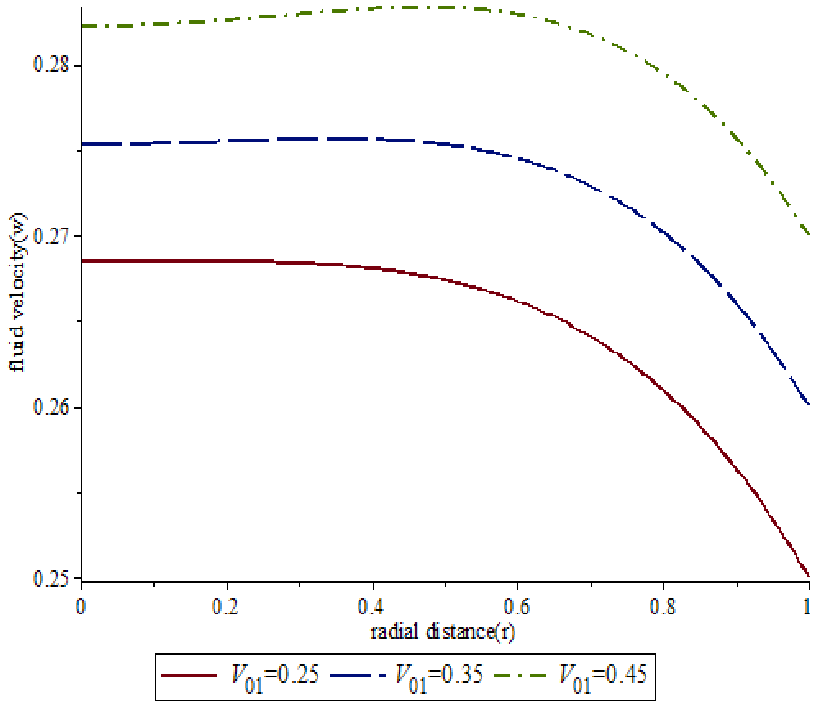

Figure 4: Variation of Velocity Profile of the Unsteady Blood Flow Model with Increasing Values of the Slip Velocity in the Radial Direction.

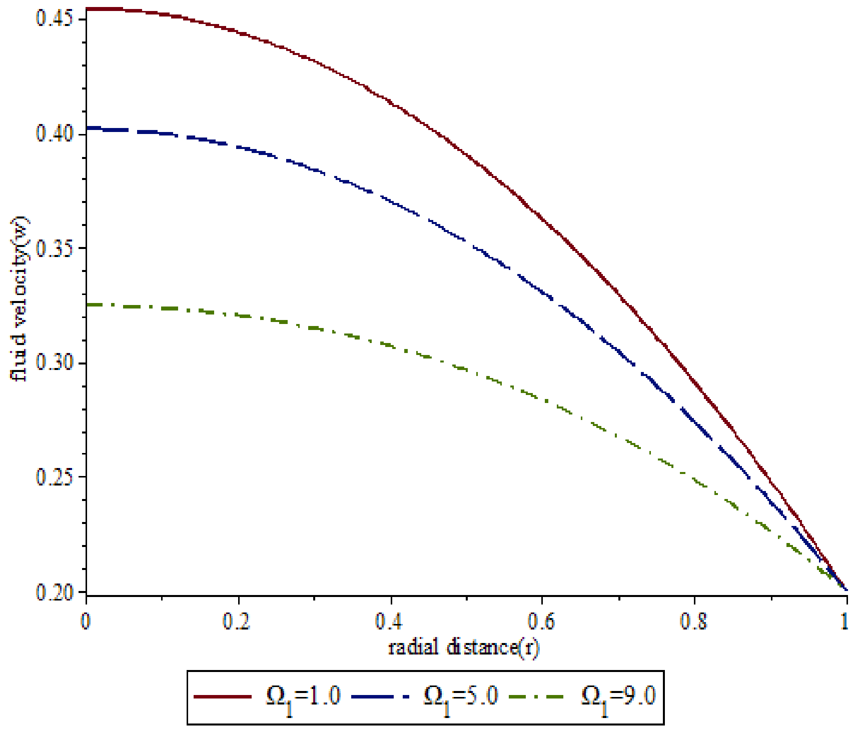

Figure 5: Variation of Velocity Profile of the Unsteady Blood Flow Model with Increasing Values of the Shear Thickening in the Radial Direction.

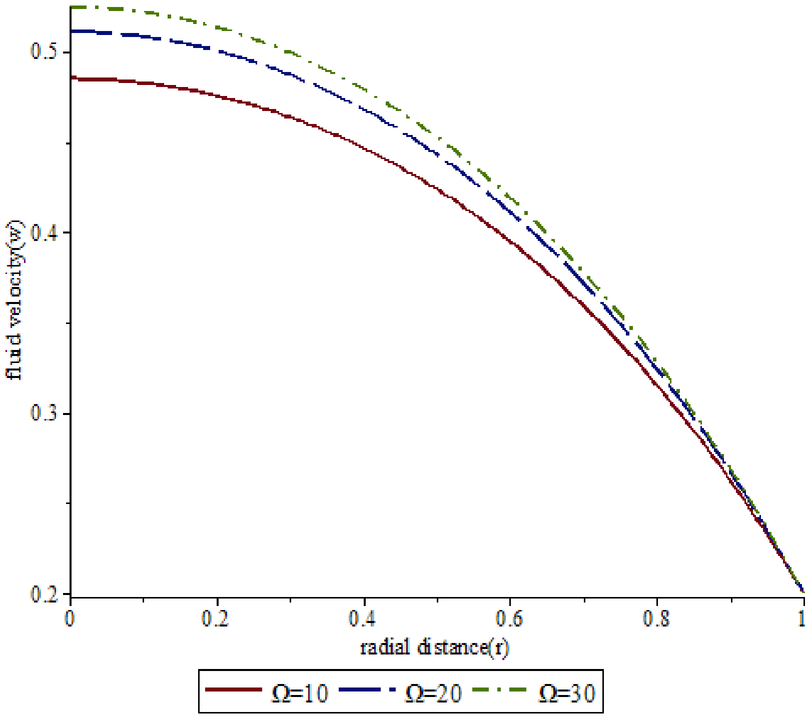

Figure 6: Variation of Velocity Profile of the Unsteady Blood Flow Model with Increasing Values of the Shear Thinning in the Radial Direction.

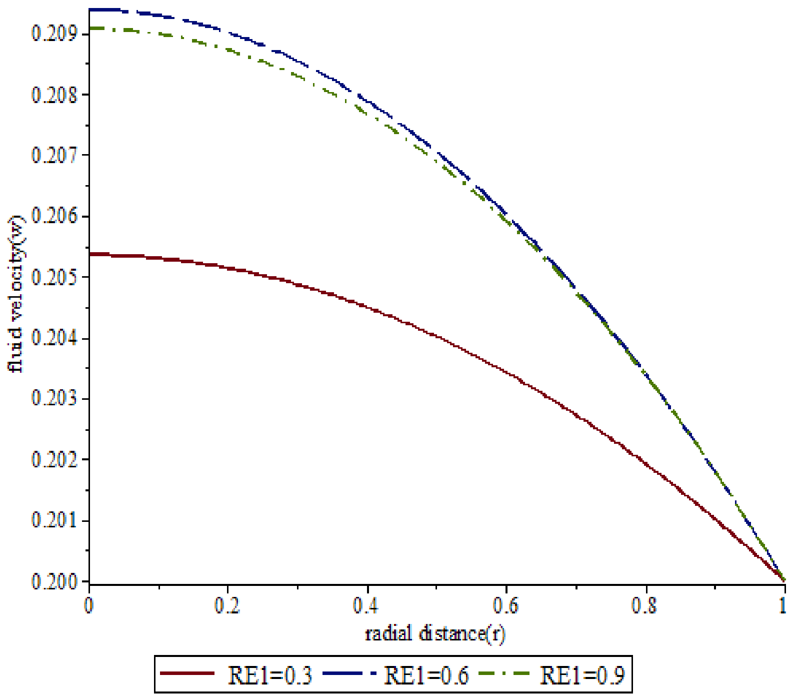

Figure 7: Variation of Velocity Profile of the Unsteady Blood Flow Model with Increasing Values of the Reynolds Number in the Radial Direction.

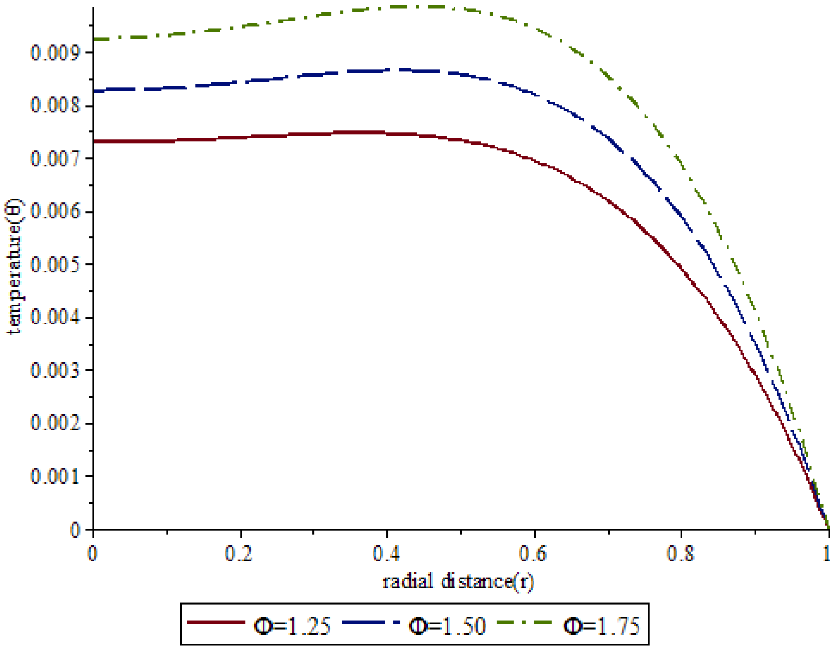

Figure 8: Variation of Temperature Profile of the Unsteady Heat Transfer with Increasing Values of the Shear Thinning in the Radial Direction.

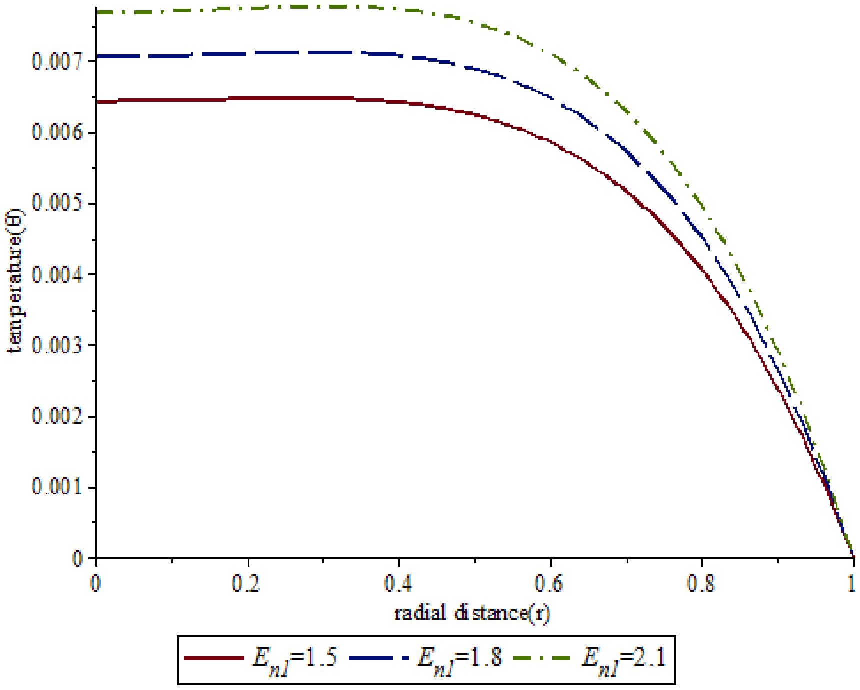

Figure 9: Variation of Temperature Profile of the Unsteady Heat Transfer with Increasing Values of the Eckert Number in the Radial Direction.

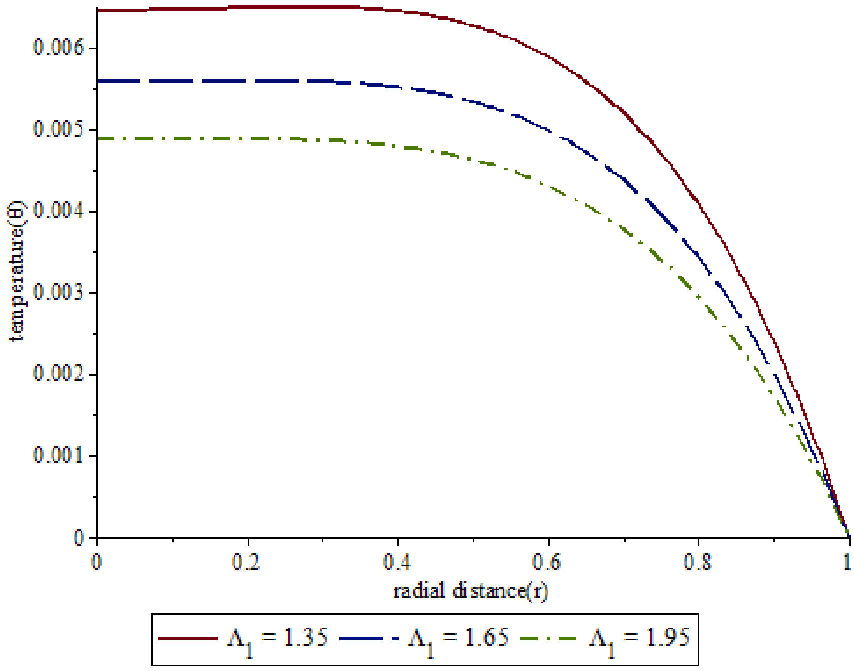

Figure 10: Variation of Temperature Profile of the Unsteady Heat Transfer with Increasing Values of the Third Grade Parameter in the Radial Direction.

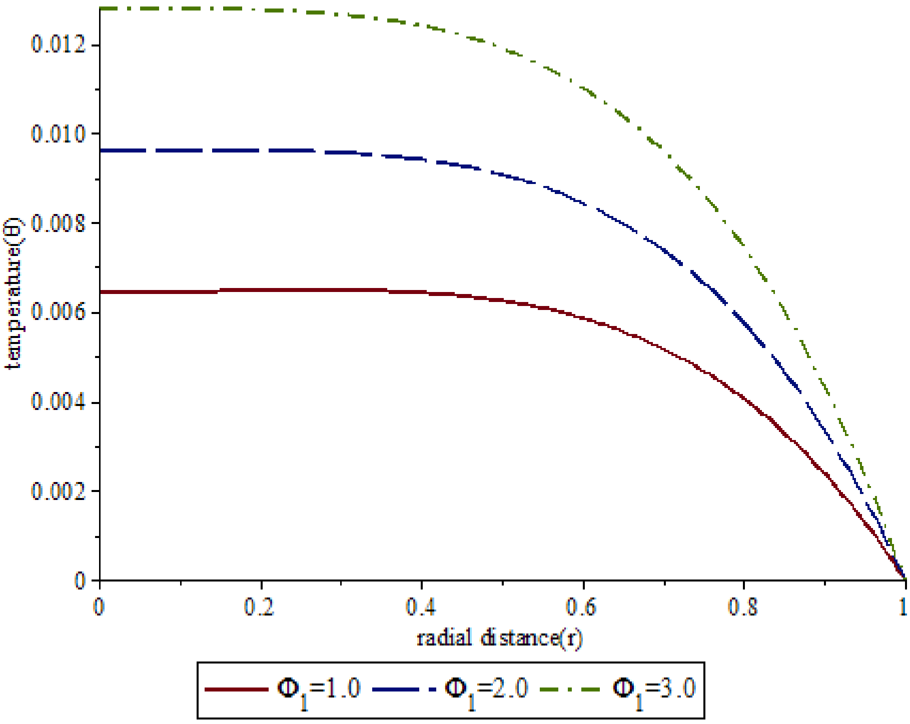

Figure 11: Variation of Temperature Profile of the Unsteady Heat Transfer without Hematocrit with Increasing Values of the Shear Thickening in the Radial Direction.

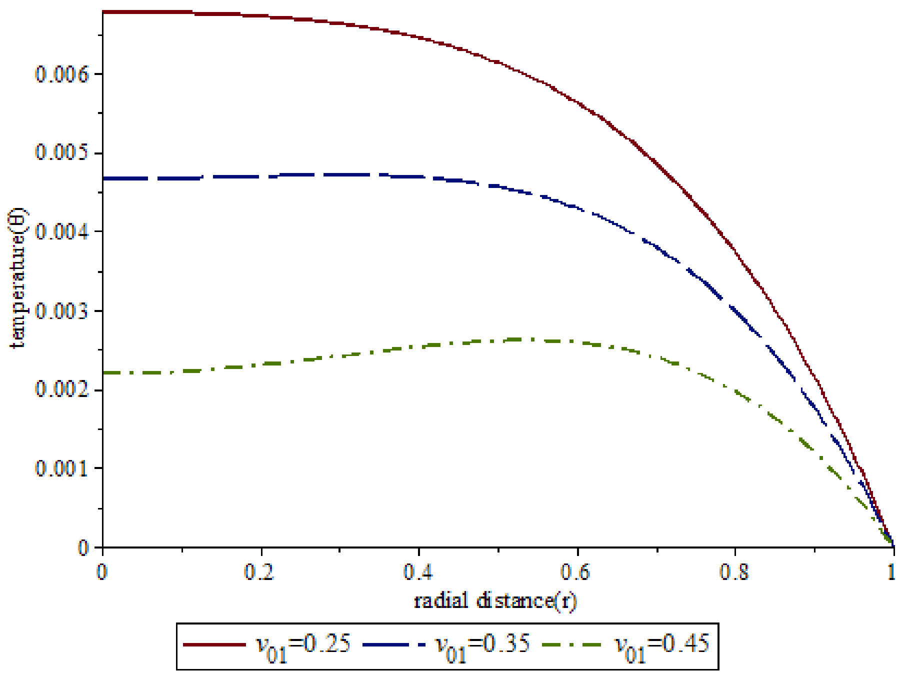

Figure 12: Variation of Temperature Profile of the Unsteady Heat Transfer with Increasing Values of the Slip Velocity in the Radial Direction.

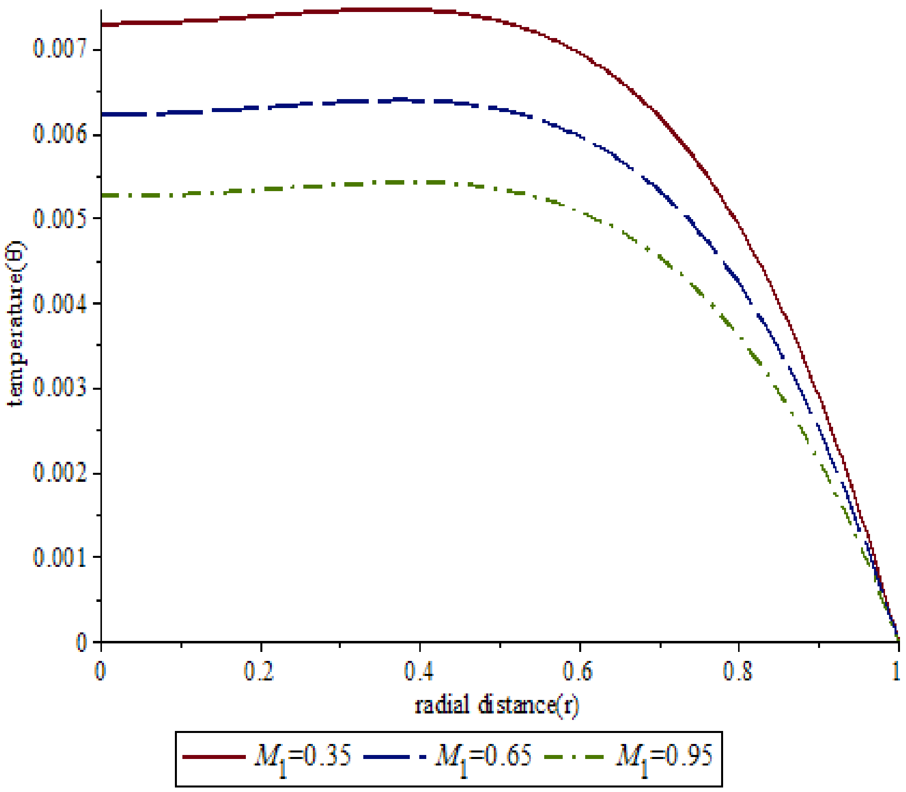

Figure 13: Variation of Temperature Profile of the Unsteady Heat Transfer with Increasing Values of the Magnetic Field Parameter in the Radial Direction.

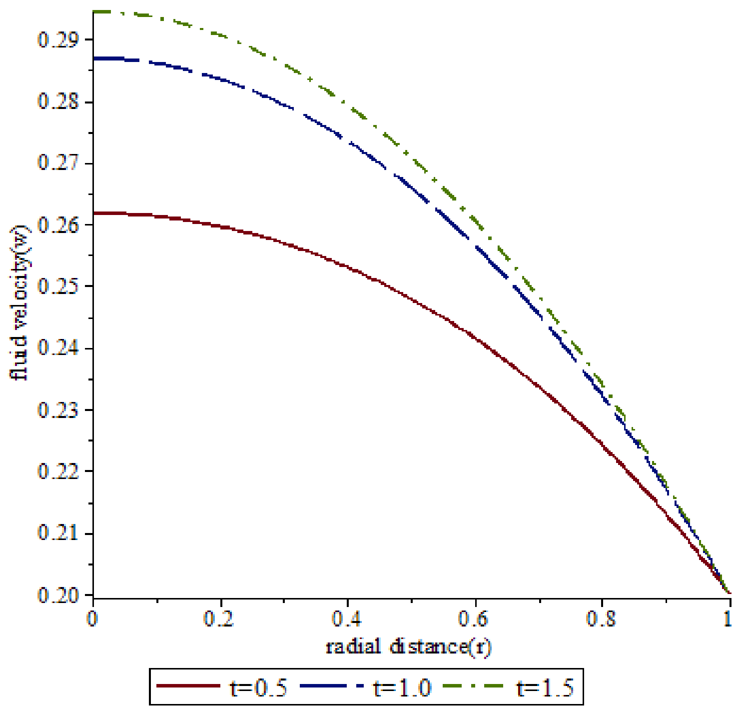

Figure 14: Variation of Velocity Profile of Unsteady Blood Flow Model with Increasing Values of Time along the Radial Direction.

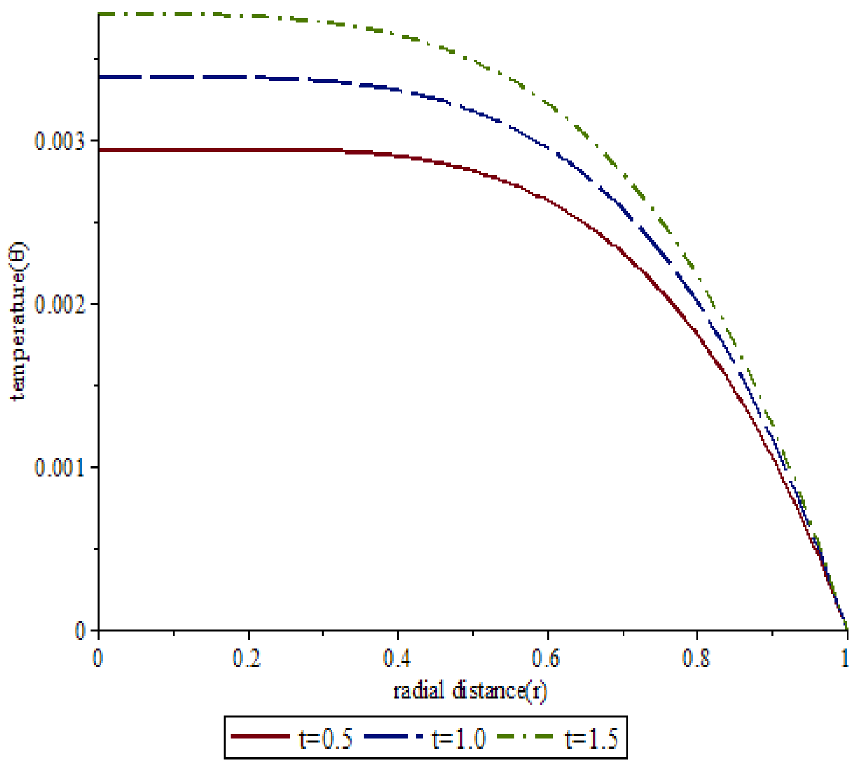

Figure 15: Variation of Temperature Profile of Unsteady Heat Transfer Model Flow Model with Increasing Values of Time along the Radial Direction.

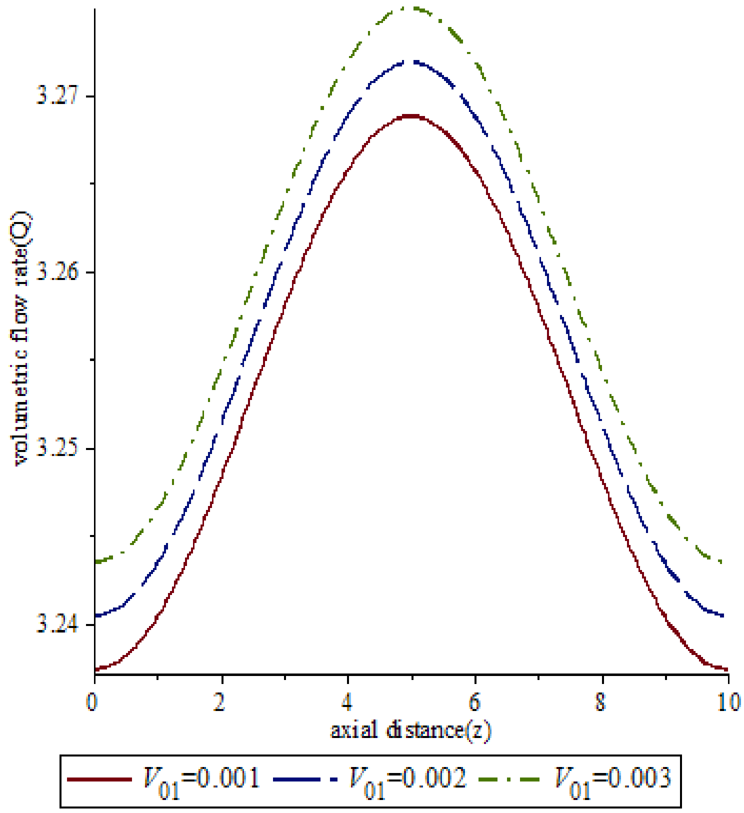

Figure 16: Variation of Volume Flow Rate of Unsteady Blood Flow Model with Increasing Values of the Slip Velocity in the Entire Stenotic Region along the Axial Direction.

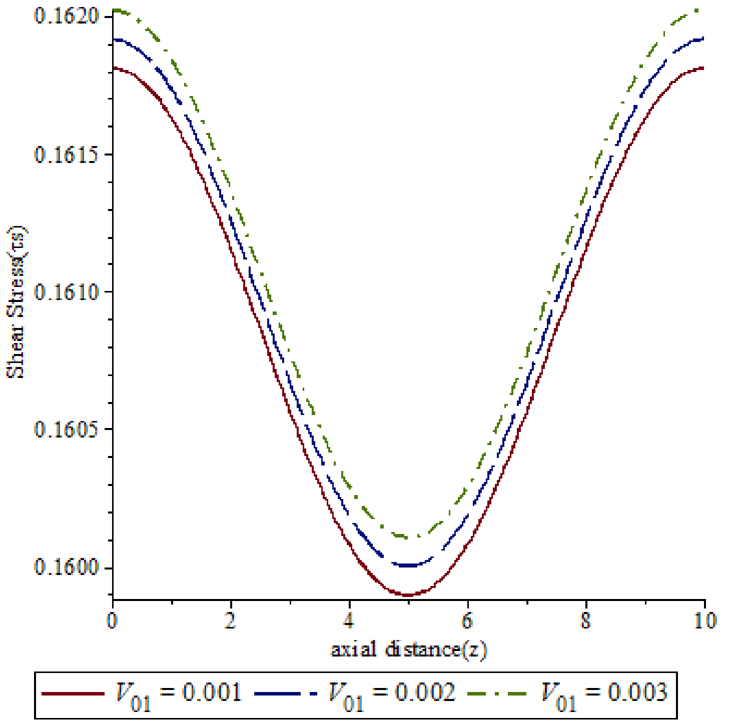

Figure 17: Variation of Wall Shear Stress of Unsteady Blood Flow Model with Increasing Values of the Slip Velocity in the Entire Stenotic Region along the Axial Direction.

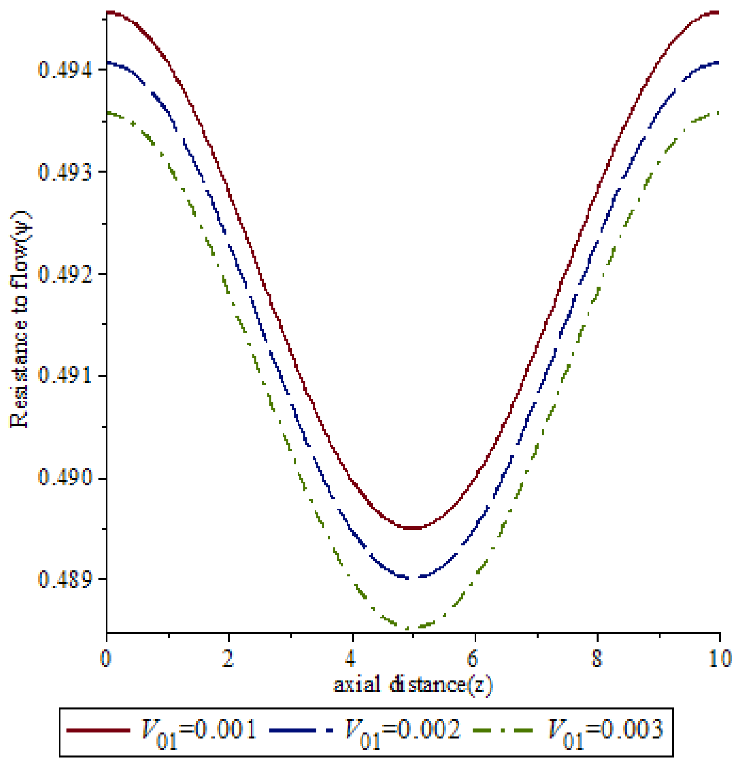

Figure 18: Variation of Wall Shear Stress of Unsteady Blood Flow Model with Increasing Values of the Slip Velocity in the Entire Stenotic Region along the Axial Direction.

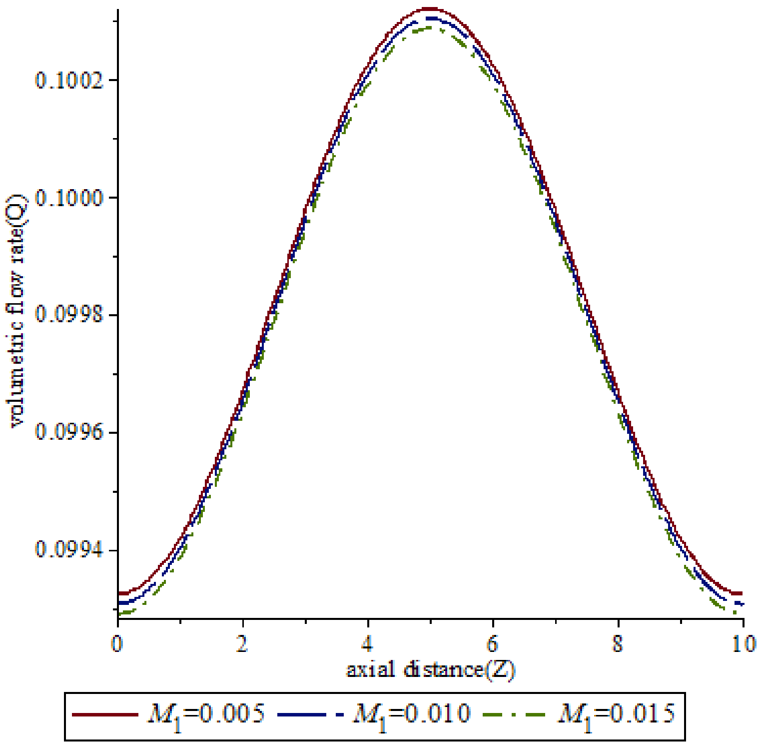

Figure 19: Variation of Volume Flow Rate of Unsteady Blood Flow Model with Increasing Values of the Magnetic Field Parameter in the Entire Stenotic Region along the Axial Direction.

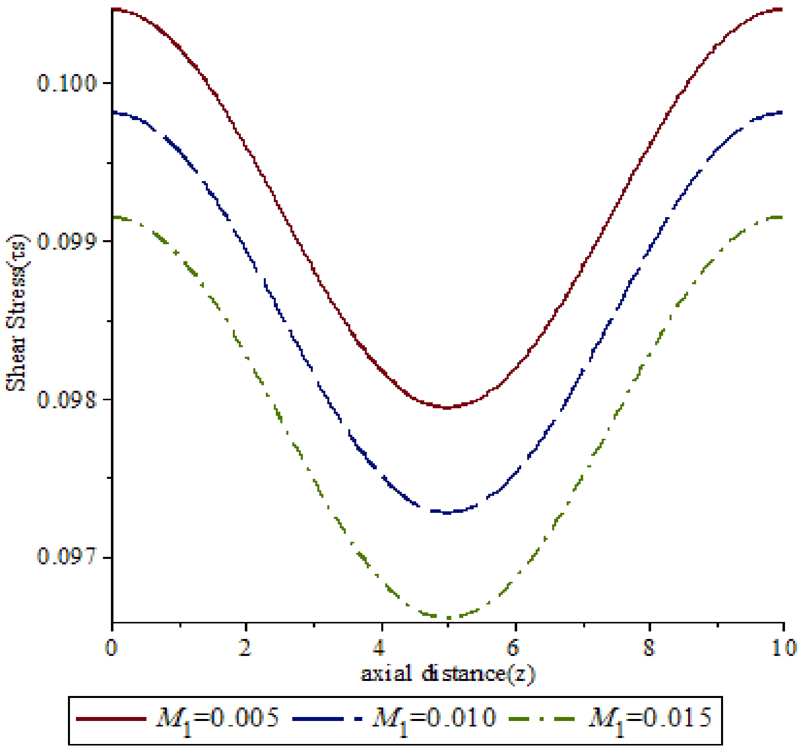

Figure 20: Variation of Wall Shear Stress of Unsteady Blood Flow Model with Increasing Values of the Magnetic Field Parameter in the entire Stenotic Region along the Axial Direction.

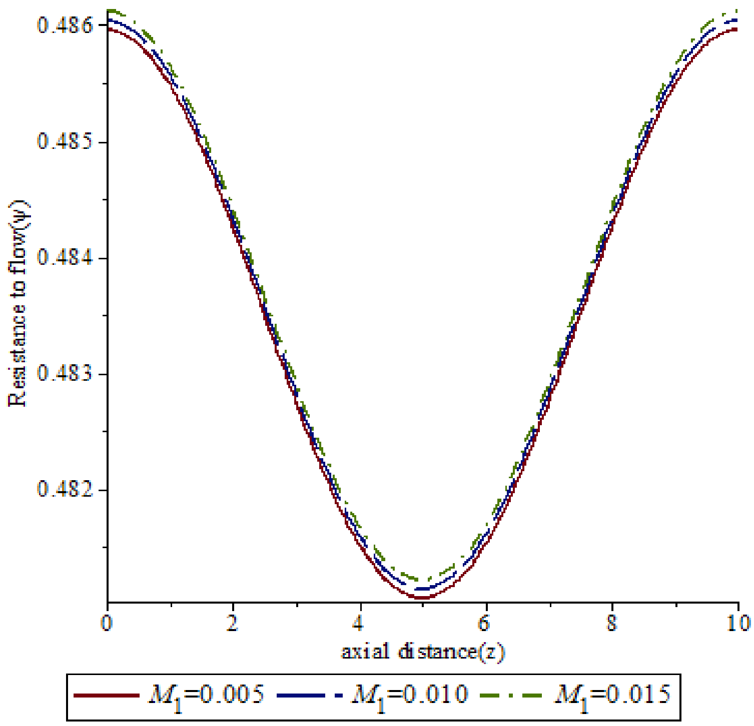

Figure 21: Variation of Resistance to Unsteady Blood Flow Model with Increasing Values of the Magnetic Field Parameter in the Entire Stenotic Region along the Axial Direction.

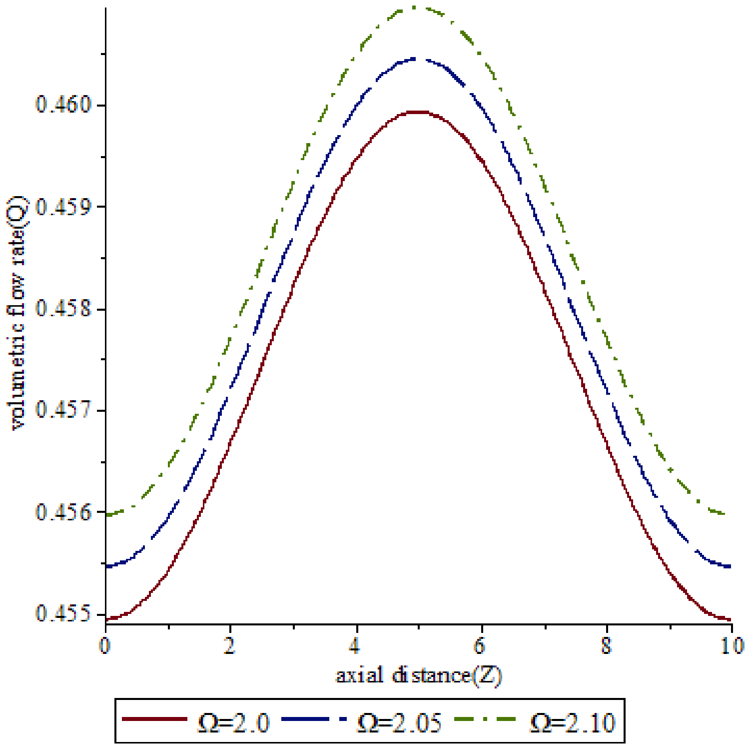

Figure 22: Variation of Volumetric Flow Rate of Unsteady Blood Flow Model with Increasing Values of the Shear Thinning in the Entire Stenotic Region along the Axial Direction.

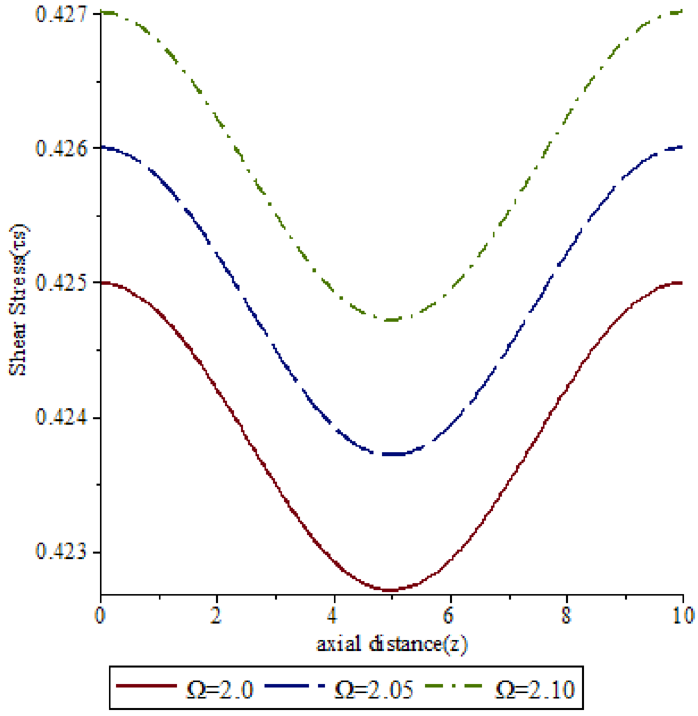

Figure 23: Variation of Wall Shear Stress of Unsteady Blood Flow Model with Increasing Values of the Shear Thinning in the Entire Stenotic Region along the Axial Direction.

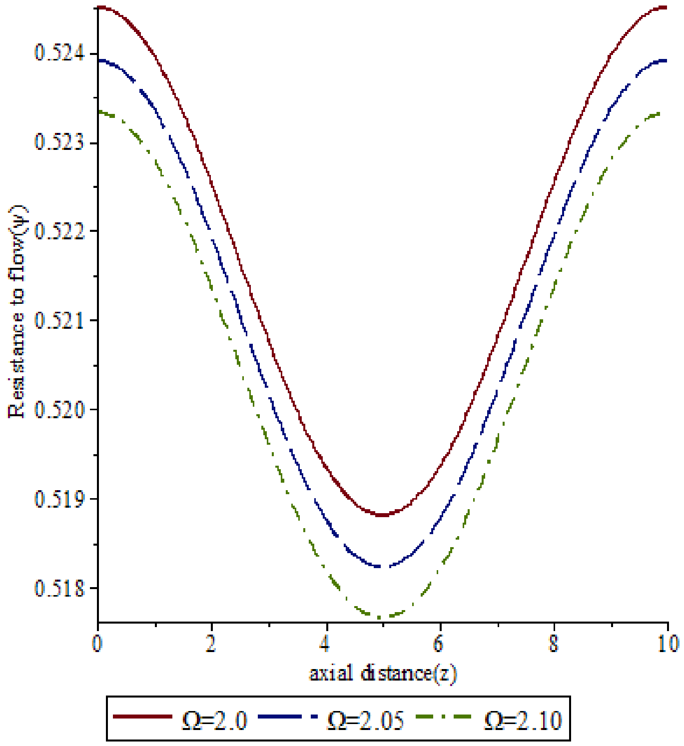

Figure 24: Variation of Resistance to Unsteady Blood Flow Model with Increasing Values of the Shear Thinning in the Entire Stenotic Region along the Axial Direction.

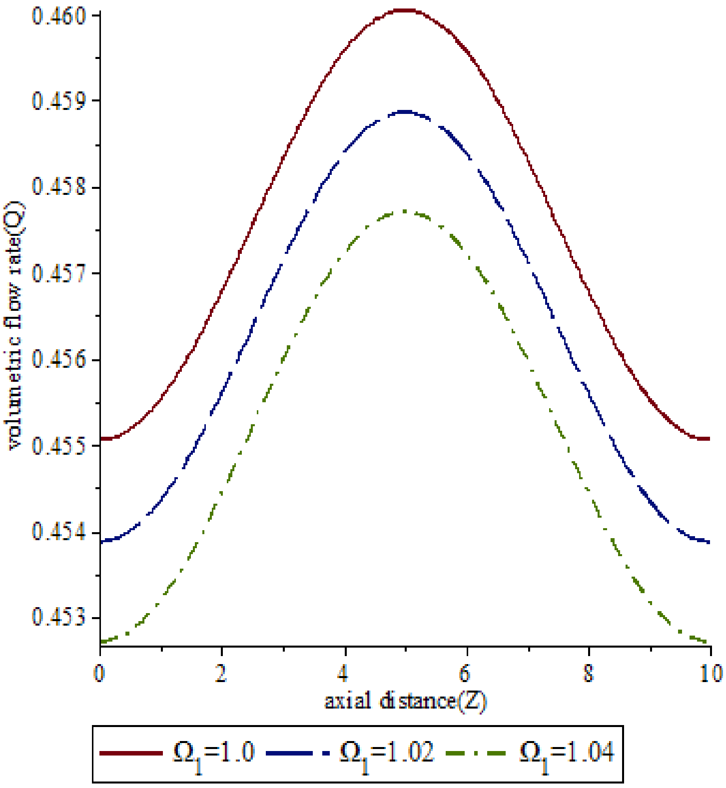

Figure 25: Variation of Volumetric Flow Rate of Unsteady Blood Flow Model with Increasing Values of the Shear Thickening in the Entire Stenotic Region along the Axial direction.

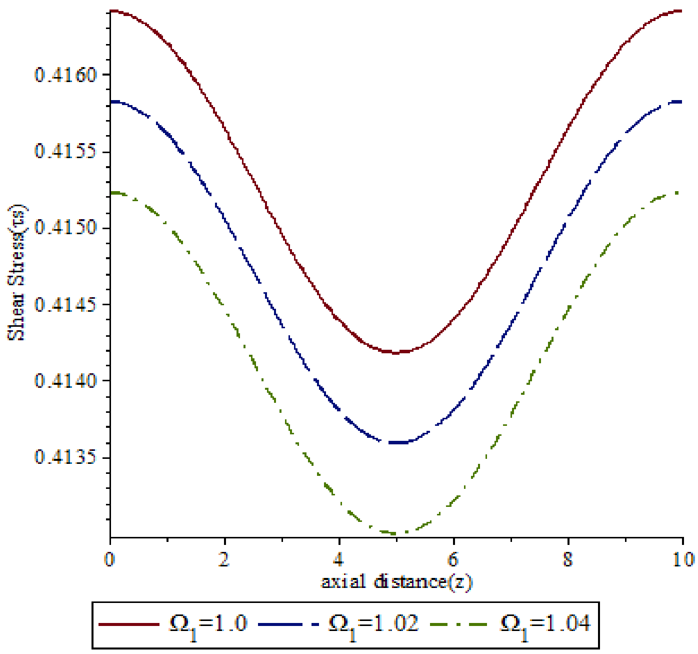

Figure 26: Variation of Wall Shear Stress of Unsteady Blood Flow Model with Increasing Values of the Shear Thickening in the entire Stenotic region along the Axial Direction.

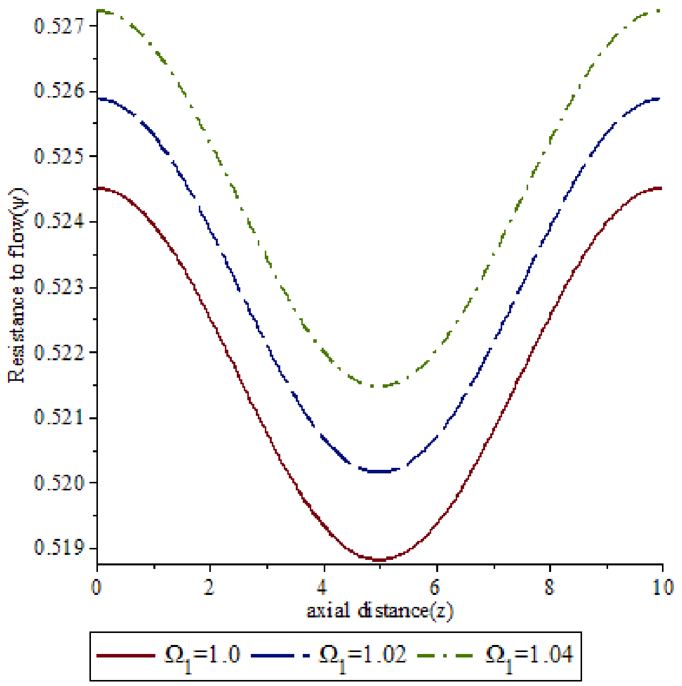

Figure 27: Variation of Resistance to Unsteady Blood Flow Model with Increasing Values of the Shear Thickening in the Entire Stenotic Region along the Axial Direction.

DISCUSSIONS OF RESULTS

In this work, we have studied the effects of velocity, magnetic field and other parameters on the flow and heat transfer characteristics by considering the blood as a third grade fluid, and the results based on the mathematical analysis indicate that an increase in slip velocity significantly leads to an increase in flow velocity, flow rate and shear stress as shown in

Figures 4, 16, 17 respectively. In the same vein, heat transfer and resistance to flow decreases as the slip velocity increases, these are depicting in figures 12 and 18 respectively. Increase in magnetic field parameter leads to decease in flow velocity, temperature profile, flow rate, shear stress but increase the resistances to fluid flow as indicated in Figures 3, 13, 19, 20 and 21, respectively. Pressure gradient and Reynolds number increase with the flow velocity as shown in Figures 2 and 7, respectively. Eckert number increase with the temperature profile as indicated in figure 9 while increase in third grade parameter leads to decrease in temperature profile as shown in figure 10.

Other important properties of blood are shear thinning and shear thickening. Increase in shear thinning leads to increase in the flow velocity, temperature profile, flow rate and shear stress but reduces the flow resistance as shown in figures 6, 8, 22, 23, and 24, respectively. Flow velocity, flow rate and shear stress decrease as shear thickening increases and these are shown in figures 5, 25, and 26, respectively while, temperature profile and resistance to flow increase with shear thickening as shown in figures 11 and 27, respectively. Finally, velocity and temperature profiles increase with time as indicated in figures 14 and 15, respectively.

CONCLUSIONS

The computational analysis of unsteady flow of blood and heat transfer through an artery with stenosis and slip conditions have been studied through this paper. The mathematical problems are solved analytically and the significant findings are summarized below:

- Velocity profile, volumetric flow rate, and shear stress increase while the temperatures profile and resistance to flow decreases with increasing values of the slip velocity.
- Velocity profile, temperature profile, volumetric flow rate, and shear stress decrease while resistance to flow increase as magnetic field parameter increases.
- Velocity profile, temperature profile, flow rate, and shear stress increase while resistance to flow decreases with increasing values of the shear thinning.
- Velocity profile, flow rate, and shear stress decrease while temperature profile and resistance to flow increase as the values of the shear thickening increases.
- Finally, increase in time positively influence both the flow and heat transfer rate.

The research analysis incorporating externally applied magnetic field is useful for the reduction of blood flow during surgery and magnetic resonance imaging (MRI). Also, the incorporating slip velocity in the constricted artery can help to reduce the blood viscosity which in effect influences the flow of blood. It is clear that the magnetic field and slip velocity are the strong parameters influencing the flow. Atherosclerosis leads to hypertension because the deposition of cholesterol or micro molecules at the arterial wall will not only reduce the regular blood supply but also cause the hardening or thickening of the blood vessel in a way that the blood vessel cannot contract easily. In view of these, the present study can also be useful to control the blood flow in disease state.

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NOMENCLATURES

w-Fluid velocity

$\bar{w}$-Dimensionless fluid velocity

t-Time component

$\bar{t}$-Dimensionless time component

r-Radial distance

y-Dimensionless radial distance

z-Axial distance

w_s-Slip velocity

V_{01}-Dimensionless Slip velocity for the unsteady blood flow

T-Temperature profile

T_w-Pipe temperature

$\bar{\theta}$-Dimensionless temperature profile

T_m-Fluid temperature

R_0-Radius of the normal artery

β_0-Magnetic Field Strength

R(z)-Radius of the artery in a stenotic region

σ-Electrical Conductivity

ψ-Resistance to flow

K-Thermal conductivity

Q-Volumetric flow rate

τ_s-Wall Shear Stress

Ƹ-Maximum height of the stenosis

L-Length of the stenosis

W-Fluid velocity

G_1-Pressure gradient for the unsteady flow

M_1-Magnetic field parameter for the unsteady flow Ω-Shear thinning for the unsteady flow Ω_1-Shear thickening for the unsteady flow

E_{n1}-Eckert number for the unsteady heat transfer

ϕ-Shear thinning for the unsteady heat transfer

ϕ_1-Shear thickening for the unsteady heat transfer

Λ_1-Third grade parameter for the unsteady heat transfer

I, Jimoh Ahmed completed my Bachelor of Technology (B.Tech) and Master of Technology (M.Tech) in industrial Mathematics both from Federal University of Technology, Akure, Ondo State, Nigeria with **First Class** and **distinction** in the years 2005 and 2013 respectively. Presently, l am waiting for my external defense of my Doctor of Philosophy (Ph. D) programme in fluid mechanics (Bio fluid) at the federal University of Agriculture, Makurdi, Benue State, Nigeria. Some of my publications where l appeared as leading author includes: Influence of Damping Coefficient and Rotatory Inertia on the Dynamic Response to Moving Load of Non-Uniform Rayleigh Beam (International Journal of Science, Engineering and Technology), Effect of Rotatory Inertial and Damping Coefficient on the Transverse Motion of Uniform Rayleigh Beam Under Moving Loads of Constant Magnitude (American Journal of Engineering Research), Dynamic analysis of Non-Uniform Rayleigh beam Resting on Bi-Parametric Subgrade under Exponentially Varying Moving Loads (Journal of Applied Mathematics and Bioinformatics), Hematocrit and Slip Velocity Influence on Third Grade Blood Flow and Heat Transfer through a Stenosed Artery (Journal of Applied Mathematics and Physics), Effect of Magnetic Field and Slip Velocity on Third Grade Blood Flow and Heat Transfer Through a Stenosed Artery (Mathematical Theory and Modelling). Hence, my research areas are dynamic of structures under moving Loads (Beam) and fluid mechanics (Bio-Fluid).I am a member of the following professional Associations: Nigeria Mathematical Society (NMS), Nigeria Association of Mathematical Physics (NAMP) and Mathematical